Behavior of sink and source defects in a one-dimensional traveling finger pattern

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We present the results of an experimental study of sink and source defects in a one-dimensional pattern of traveling fingers that form at a driven fluid-air interface. We find that sinks and sources behave differently: Sinks separate regions of differing wave number and move smoothly so as to keep the phase difference across the sink fixed. They are transient objects which are eventually destroyed at the boundaries of the experiment or by collision with a source. Sources, on the other hand, are long lived. They are symmetric and stationary on average, although individual sources move erratically and do not display the phase-matching behavior of the sinks.

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I. INTRODUCTION

The patterns which form in driven nonlinear systems and the instabilities which lead to them have been the focus of considerable theoretical and experimental interest in recent years [1]. In particular, systems which display patterns in one spatial dimension have been of interest because their simplicity relative to three-dimensional systems allows for comprehensive quantitative comparison between experiment and theory [1]. When the pattern is a traveling wave, source and sink defects, which separate regions in which the wave travels in opposite directions, can exist. These defects can have an important influence on the behavior of the pattern as a whole.

Much of the work on traveling-wave patterns to date has focused on single-domain regions with no defects [1], but sources and sinks have been observed in a variety of experimental systems. These include electroconvection in liquid crystals [2], convection driven by a heated wire suspended in a fluid [3], convection in binary mixtures [4], Taylor-Dean flow [5], and the printer's instability [6,7]. Sources and sinks have been studied theoretically by several groups [8–11]. Much of this work is reviewed in Refs. [1] and [12].

Recently van Hecke *et al.* [12] have done extensive theoretical and computational studies of defects in coupled complex Ginzburg-Landau equations, which model competing one-dimensional traveling waves close to a supercritical Hopf bifurcation from a spatially uniform state. They showed that in general a discrete set of sources exists, and suggested that a unique source should be observed experimentally. Above a certain value of the system's control parameter, sources are stable, stationary, and symmetric, that is, they emit waves with the same wave number on either side. Sinks, on the other hand, form a continuous family and are not generally stationary; traveling sinks have a mismatch in wave number on either side and therefore have a nonzero velocity [12].

Goldstein *et al.* [13,14] studied sources and sinks separating regions of counterpropagating traveling waves in a Ginzburg-Landau model of a parity-breaking transition. A phase instability, analogous to the Eckhaus instability [15], led to the production or destruction of traveling waves in the core of the defects. They studied defects with the same wave number on either side; the defects were stationary by symmetry. They found two regimes: one in which both sources and sinks are stable and one in which only one type of defect (determined by the sign of a coupling constant in the model) is stable.

Alvarez *et al.* performed experiments on convection patterns driven by a heated wire [3]. They observed patterns consisting of two sources emitting different wave numbers flanking a single moving sink. The sink moved such that the phase difference across it remained fixed. Both the appearance of multiple sources and the phase matching motion of the sinks appear to be inconsistent with the coupled complex Ginzburg-Landau equation model of Ref. [12], the latter because the phase matching condition involves fast length scales which are not included in the Ginzburg-Landau description.

It is straightforward to show that this phase matching motion implies that the velocity v_{sink} of the sink is given by

$$v_{\mathrm{sink}}=\frac{\omega_R-\omega_L}{k_R+k_L}, \tag{1}$$

where ω and k are the frequency and wave number of the traveling waves coming into the sink, and the subscripts R and L refer to the right- and left-moving waves, respectively. Note that since $\omega=v_{\mathrm{ph}}k$, where v_{ph} is the phase velocity of the traveling waves, then if v_{ph} is the same on both sides of the sink, the sink velocity v_{sink} is proportional to $\delta k=k_R-k_L$, the difference in wave number across the defect.

In this paper we study the behavior of source and sink defects in the system known as the ''printer's instability'' [6,7,17–19]. The system consists of a thin layer of oil confined in a narrow diverging gap between two acentric horizontal cylinders. The oil-air interface is driven out of equilibrium by rotating one or both of the cylinders, and a one-dimensional pattern of fingers forms along the length of the interface. The formation and dynamics of patterns in the printer's instability have been studied in some detail by Rabaud and co-workers [6,16,17] and by Pan and de Bruyn [7,18,19]. Rabaud *et al.* mapped out the dynamical phase diagram of this system as a function of the rotation velocities of the two cylinders [6,16]. When a single cylinder rotates above a certain critical speed, the interface becomes unstable to a pattern of stationary, symmetric fingers. When the second cylinder is then set into counter-rotation, a parity-breaking transition [20] occurs at which the fingers lose their left-right reflection symmetry and start to travel [7,18,19]. If regions of both left- and right-traveling fingers coexist, they will be separated by defects [16].

II. EXPERIMENT

Our experimental system, which has been described in detail elsewhere [7,18,19], consists of two horizontal cylinders, mounted one inside the other. The axes of the cylinders are accurately parallel, but offset vertically, so that the gap between the cylinders is smallest at the bottom. The inner cylinder was made of Delrin and had a radius of 38.7 mm and a length of 202 mm. The outer cylinder was made of Plexiglas and had a radius of 66.7 mm and length of 210 mm. In this work, the minimum gap thickness between the cylinders was $b_0 = 0.5 \pm 0.05$ mm. A quantity of silicone oil (viscosity 0.525 g/cm s, surface tension 19.4 g/s^2, and density 0.963 g/s^3 [21]) sufficient to keep the space between the cylinders filled was poured into the gap. The cylinders could be independently rotated about their axes under computer control. The oil-air interface at the front of the apparatus was monitored with a video camera interfaced to a computer.

When the inner cylinder was rotated so that the bottom of the cylinder moved towards the camera, while the outer cylinder was held fixed, stationary fingers appeared on the oil-air interface at an inner cylinder velocity v_i of $v_{ic} = 197$ mm/s. The fingers appeared with finite amplitude and, if the inner cylinder velocity was then decreased, disappeared at a lower value of v_i. The bifurcation to fingers is thus subcritical [22]. We define $v_i^* = (v_i - v_{ic})/v_{ic}$; the experiments reported here were performed in the range $0.22 \leqslant v_i^* \leqslant 0.64$. Any slight misalignment of the cylinders leads to spatial variations in the wave number of the pattern, as well as to a slow drift of the fingers. We ensured that the cylinder axes were accurately parallel by measuring the uniformity of the finger pattern and confirming the absence of drift.

With v_i fixed, the outer cylinder was then set into counter-rotation. Traveling fingers appeared via a supercritical parity-breaking bifurcation [13,18] at an outer cylinder velocity $v_o = v_{oc}$. We found $v_{oc} = 1.33 \pm 0.01$ mm/s, independent of v_i within our measurement uncertainty. We also define $v_o^* = (v_o - v_{oc})/v_{oc}$; this is the relevant control parameter for these experiments. Images of these traveling fingers are shown, for example, in Figs. 1 and 5 of Ref. [19].

We found that source and sink defects could be created by two methods. First, they appear naturally in the traveling finger pattern when the rotation velocity of one or the other of the cylinders is suddenly changed. Under these circumstances, the oil-air interface breaks up into several regions of left- and right-traveling waves separated by defects. Most of these defects are short-lived transients, which annihilate each other as the system approaches a steady state. The steady state pattern normally consists either of a single domain of fingers traveling either left or right, or of two opposite domains separated by a single long-lived source. An image of a source is shown in Fig. 4 of Ref. [7]. We were unable to make long-lived sink defects by this method. Sinks created by a sudden jump in v_i or v_o were always transients, which did not survive long enough for quantitative study.

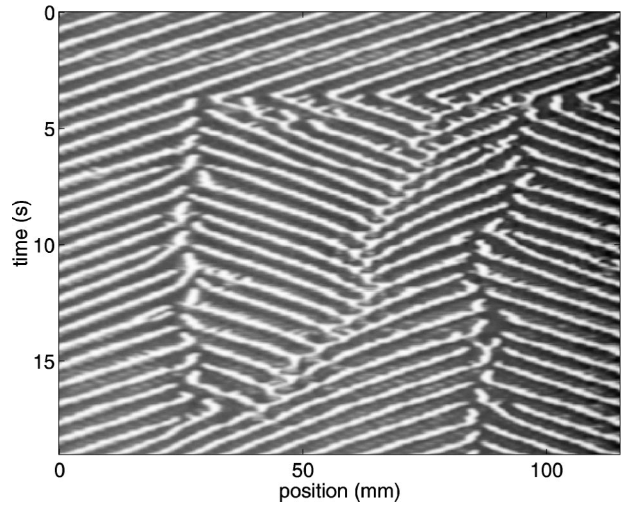

FIG. 1. A space-time image of the fingering pattern at $v_o^* = 15.0$, illustrating the formation and dynamics of source and sink defects. Two sources and a sink form at $t \approx 4$ s. The sources are approximately stationary, while the sink travels leftward and annihilates with one of the sources at $t \approx 17$ s.

Sources and sinks could also be created by physically perturbing the steady state pattern. A rubber wiper attached to a rod was used to locally disturb the layer of fluid coating the inner cylinder. This in turn perturbed the finger pattern on the oil-air interface. This method produced both sources and sinks for a range of v_o^*. Close to the onset of the traveling wave state, sinks could not be created at all, although sources could. However for $9.3 < v_o^* < 16.2$ it was possible to create sinks which, while still ultimately transients, survived long enough to be studied quantitatively. The experiments reported here were performed at $v_o = 22.75$ mm/s, that is, at $v_o^* = 16.1$. One would not expect any theoretical model based on a small-amplitude expansion to be valid so far above the parity breaking transition.

III. RESULTS

Figure 1 is a spacetime image which illustrates the dynamics of three defects. This image was constructed by periodically recording a single video line through the finger pattern, then stacking these line images vertically. Thus in Fig. 1, position along the length of the experiment runs horizontally while time runs vertically from top to bottom. Initially the pattern is a uniform left-moving traveling wave. Suddenly three defects form: a sink flanked on each side by a source. In this case the defects appeared to form spontaneously, presumably in response to an unobserved perturbation at the end of the apparatus. The two source defects are essentially stationary, while the sink travels to the left. When it meets the leftmost source, the two annihilate and only the right hand source remains. In general, we observed that isolated sources were approximately stationary and survived for

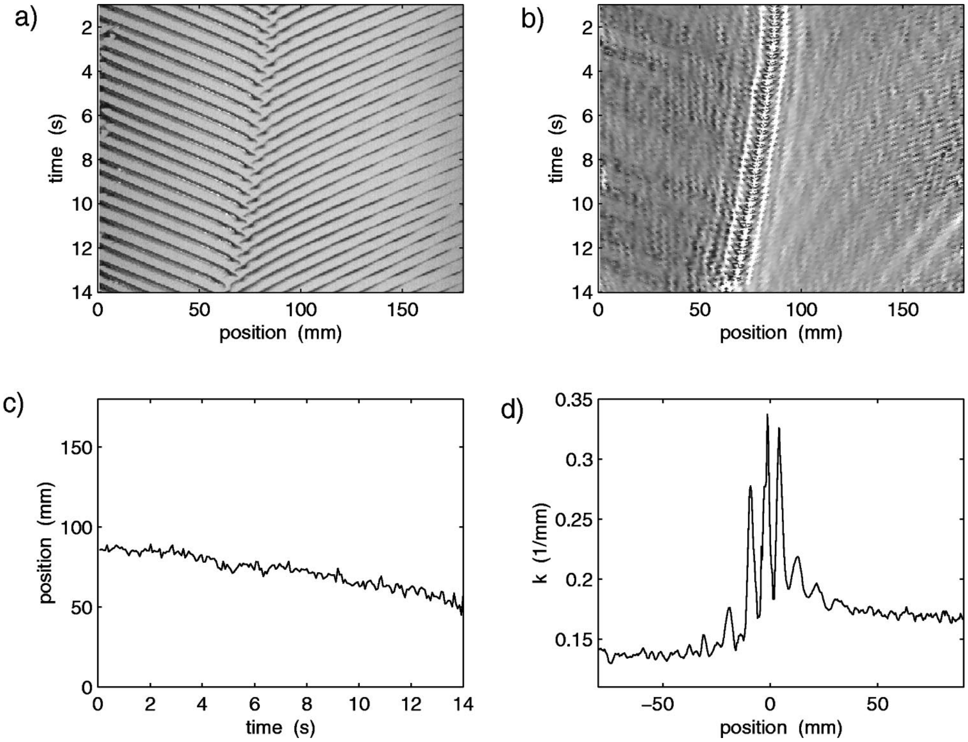

FIG. 2. (a) A spacetime image of a sink defect at $v_i^* = 0.51$, $v_o^* = 16.1$. (b) The local wave number field $k(x,t)$ calculated from the image in (a). Darker shades indicate lower k. The high values of k at the defect are used to identify the position of the defect, which is plotted in (c). (d) The time-averaged wave number as a function of position $\langle k(x) \rangle$ determined from (b). $\langle k(x) \rangle$ is higher on the right side of the sink than on the left. In (d), position is measured relative to that of the defect.

the duration of the experiment. Isolated sinks were always transients which survived for at most a few minutes. Isolated sinks traveled through the pattern and were eventually destroyed at the ends of the cylinder. When a source and a sink were created together, the sink would travel at a constant speed towards the source and the two would annihilate when they collided, as in Fig. 1.

The ends of the cell behaved as passive boundaries, serving as sources or sinks of traveling fingers in response to the demands of the pattern. Traveling fingers that reached the ends of the system were absorbed with no observable reflection.

We analyzed spacetime images to extract the velocity of the defects, the spatial wave number k, and the temporal frequency ω of the pattern as a function of position and time. k and ω were found using a simple variant of the method of Ref. [23]. The spacetime image was first bandpass filtered in Fourier space to remove all but the fundamental spatial and temporal frequencies. Appropriate derivatives of the filtered real-space image then yield both ω and k [23]. Figure 2 shows the results for a sink moving towards the left. The unfiltered spacetime image is shown in Fig. 2(a); the fingers on both sides of the defect move with their phase velocity towards the sink. Figure 2(b) shows the corresponding wave number field $k(x,t)$. The wave number is higher on the right of the defect than on the left, as indicated by the grayscale shading; this difference is more obvious in Fig. 2(d). The method used to determine k gives anomalously high values at the defect. We turn this to advantage by using an edge detection algorithm to locate the defect based on these high k values. The position of the sink determined in this way is plotted as a function of time in Fig. 2(c), and the slope of this graph gives the sink's velocity. Figure 2(d) shows the wave number averaged over the time covered by Fig. 2(b). This quantity is plotted as a function of position relative to the position of the moving defect. This figure again shows that the wave number of the pattern away from the sink is larger on the right than on the left. The sink here moves towards the region of lower k, as was the case with all sinks we observed.

Defects are properly classified as sources or sinks in terms of the group velocity v_g of the traveling waves, not the phase velocity [12]. Modulations in k can be seen moving towards the sink in Fig. 2(b), particularly at the lower right of the image. Corresponding modulations in the finger spacing can be seen in Fig. 2(a). Since these modulations travel at the group velocity, this demonstrates that v_g is in the same direction as, but somewhat lower than v_{ph}, and confirms our intuitive identification of the defect in Fig. 2 as a sink.

Analogous results for a source defect are shown in Fig. 3. This source drifts rather slowly to the right, and Fig. 3(d) shows that the values of $\langle k(x) \rangle$ on the two sides of the source are almost the same. For this defect the wave number is on average slightly larger on the left, and on average the source moves to the right. As before, modulations in wave number can be seen in the upper right of Fig. 3(b), indicating that v_g is again in the same direction as, but slower than v_{ph}.

The motion of sources was qualitatively different from that of sinks. Figures 2(a) and 2(c) show that sinks move smoothly, with left-moving and right-moving fingers coming into the defect alternately, very similar to a zipper. On the other hand, the motion of the source in Fig. 3 is irregular, consisting of periods when it is stationary, sudden jumps in one direction or the other, and short periods of relatively steady drift. The emission of fingers from the source is also not regular, and there is considerable variation in the wave number of the emitted fingers very close to the defect.

The motion of sink defects is not perfectly regular, however. Occasionally the sink will undergo a ''phase jump'' at

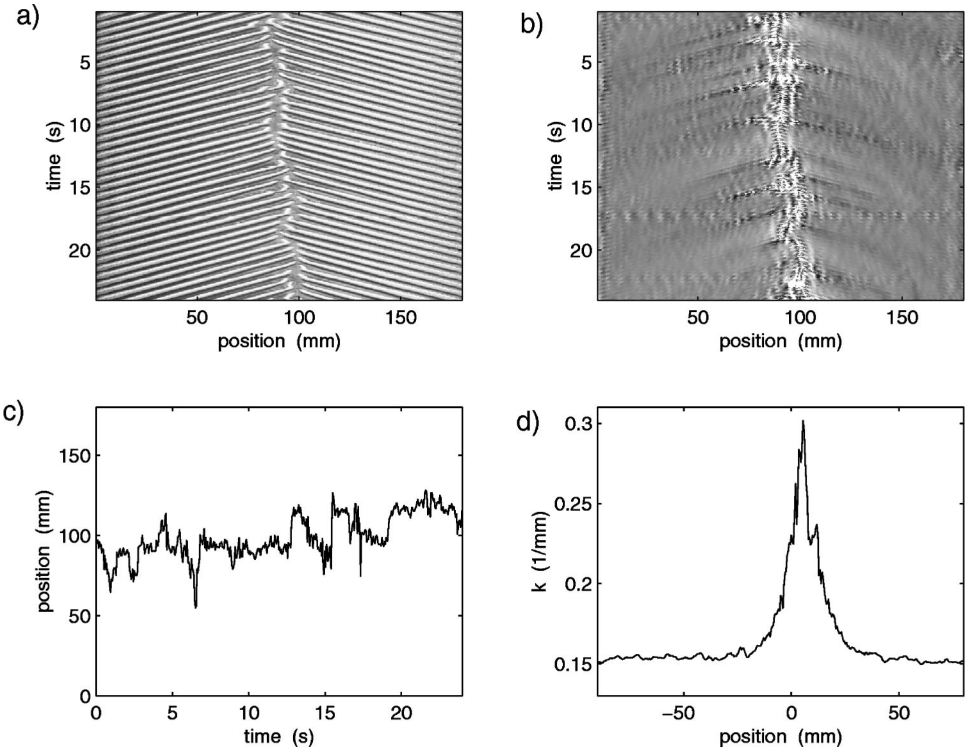

FIG. 3. (a) A spacetime image of a source defect at $v_i^* = 0.51$, $v_o^* = 16.1$. (b) The local wave number field $k(x,t)$ calculated from the image in (a). Darker shades indicate lower k. (c) The position of the defect extracted from (b). (d) The time-averaged wave number $\langle k(x) \rangle$ determined from (b). $\langle k(x) \rangle$ is approximately equal on the two sides of the source. In (d) position is measured relative to that of the defect.

which the defect will move one wavelength left or right, absorbing an extra finger in the process. This is illustrated in Fig. 4. Here we show a sink which is generally moving steadily to the left. At the times indicated by the arrows, however, the sink moves one wavelength to the right as it absorbs two left-moving fingers in a row. In many cases these jumps seemed to be triggered by perturbations to the pattern propagating in to the defect from the boundary of the experiment. These jumps have a large effect on the average velocity of a sink—in the case shown in Fig. 4, the sink's steady drift is to the left, but because of the jumps, its average velocity over the time of the image is to the right. Periods containing these jumps were excluded from the analysis when the wave number of the pattern and the velocity of the sinks was determined.

Figure 5 shows an example of the Eckhaus instability of the finger pattern [7,15]. When the local wavelength of the pattern becomes too large, a new finger forms to bring the pattern's wavelength back into the range of stability. This process has been studied in this system by Pan and de Bruyn [7]. Since this instability modifies k locally, regions affected by this instability were also excluded from the analysis.

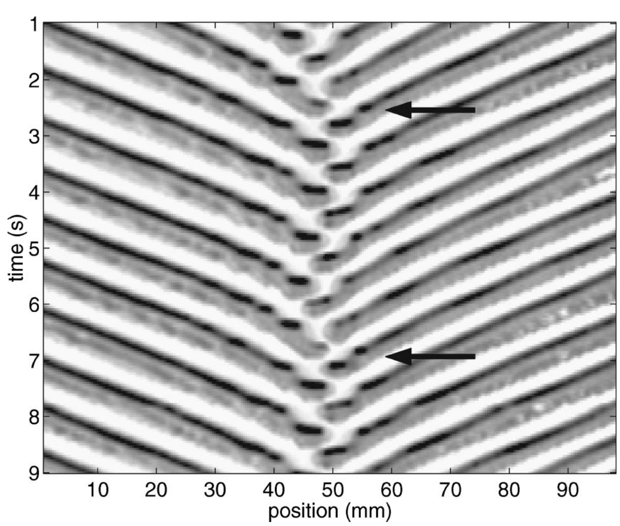

FIG. 4. A spacetime image of a sink at $v_i^* = 0.22$ and $v_o^* = 16.1$ showing two phase jumps, accompanied by a shift to the right in the position of the sink, at the times indicated by the arrows.

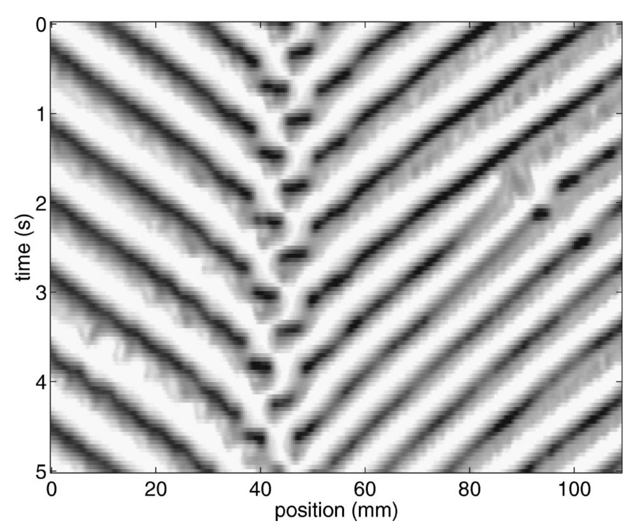

FIG. 5. A spacetime image of a sink at $v_i^* = 0.34$ and $v_o^* = 16.1$. The finger pattern on the right-hand side of the defect undergoes an Eckhaus instability at $t \approx 2$ s, leading to a change in the local wave number of the pattern.

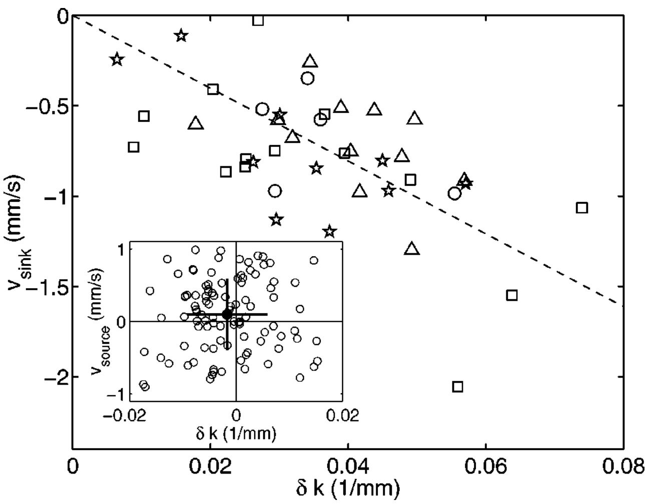

FIG. 6. The velocity of sink defects as a function of wave number mismatch for $v_o^* = 16.1$ and several inner cylinder speeds. Stars: $v_i^* = 0.28$; triangles: $v_i^* = 0.46$; squares: $v_i^* = 0.58$; circles: $v_i^* = 0.64$. The dashed line is a fit through the origin to the plotted data. The inset shows the velocity of source defects for $v_o^* = 16.1$ and a variety of inner cylinder speeds in the range $0.22 \leqslant v_i^* \leqslant 0.64$. The solid circle shows the average values of v_{source} and δk, with the error bars indicating plus and minus one standard deviation.

Figure 6 shows the velocity of sink defects v_{sink} as a function of the wave number mismatch δk across the sink. Data for several different values of the inner cylinder speed v_i^* are shown; measurements at other values of v_i^* overlap those shown within the scatter. In all cases $v_o^* = 16.1$, which is far above the onset of the traveling finger state. Figure 6 shows that v_{sink} is proportional to δk within the scatter. The dashed line in Fig. 6 is a least squares fit to the data shown, constrained to pass through the origin; its slope is $-20 \pm 4\ \text{mm}^2/\text{s}$. An unconstrained fit to a straight line was consistent with this result within its uncertainty. The scatter in the data is probably due to uncertainties involved in the extraction of the data from the spacetime images, as well as to experimental noise. All sinks we observed had a negative (i.e., leftward) velocity. This may be due to small experimental asymmetries, or to the possible presence of a large-scale flow along the length of the cylinders [16] driven by flows at the ends of the apparatus.

Source velocities v_{source} are shown for several values of v_i^* as a function of δk in the inset of Fig. 6. Again $v_o^* = 16.1$. In this case the data are clustered around $(\delta k, v_{\text{source}}) = (0,0)$. No systematic dependence of v_{source} on either δk or v_i^* was observed. The mean values of δk and v_{source} are shown on the inset to Fig. 6; both are zero within our uncertainty. This suggests that the observed motion of the sources is due not to any fundamental physics of the pattern, but rather to experimental noise or uncontrolled local perturbations of the defect. This is consistent with the erratic nature of the source motion illustrated in Fig. 3.

The spacetime image of Fig. 2 indicates that sinks move to keep the phase difference across the defect constant. If so, then Eq. (1) should apply. Figure 7 is a test of this. Here we

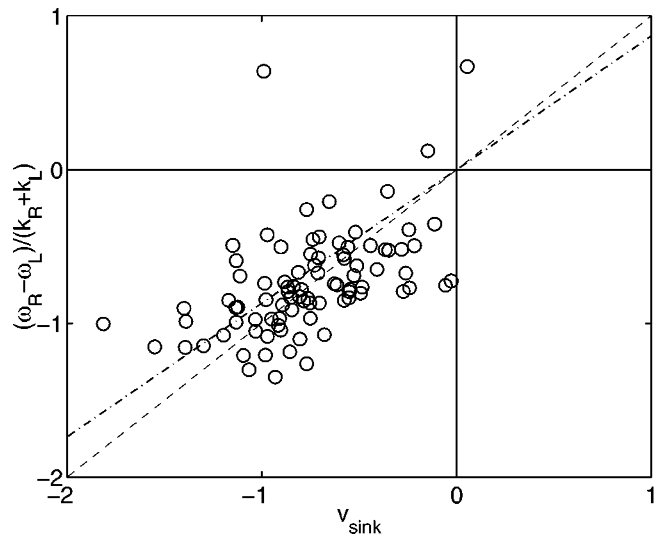

FIG. 7. A plot of $(\omega_R - \omega_L)/(k_R + k_L)$ vs v_{sink} for a number of sinks at $v_o = 16.1$, $0.22 \leqslant v_i^* \leqslant 0.64$. The dashed line through the origin has a slope of one and represents the equality predicted by Eq. (1) for phase matching motion. A fit to our data constrained to go through the origin gives the dash-dotted line, which has a slope of 0.87 ± 0.12.

have plotted $(\omega_R - \omega_L)/(k_R + k_L)$ against v_{sink} for all of our data. The dashed line has a slope of 1 and represents the equality predicted by Eq. (1). Our data are consistent with this prediction. The dash-dotted line is a fit to the data constrained to go through the origin; it has a slope of 0.87 ± 0.12. Our results are therefore in agreement with the phase matching equation within the experimental scatter.

IV. DISCUSSION

van Hecke *et al.* [12] have recently studied source and sink defects in coupled complex Ginzburg-Landau equations. Our experimental system is somewhat different from the theoretical system they studied. Their approach is expected to be valid close to the onset of traveling waves at a supercritical Hopf bifurcation. In our case, the traveling waves arise due to a parity-breaking bifurcation, and our experiments were performed far above onset. Because of these differences, any attempt to compare quantitatively the experimentally observed behavior with that of the model of Ref. [12] would be unjustified. Nonetheless, there are some qualitative similarities—and some differences—between experiment and theory which bear discussion.

van Hecke *et al.* found that sources were stationary and symmetric, that is, that they separated regions of oppositely traveling waves with the same *k*. In our experiments, sources moved irregularly, but on average had zero velocity and the same wave number on either side, consistent with the theoretical behavior. Sinks in Ref. [12] separated regions of different *k* and traveled, as do ours. We observe that our sinks normally move to achieve phase matching, that is, so that the phase relationship between the incoming fingers on the two sides of the defect remains fixed. Similar defect dynamics was observed for sources and sinks in convection experiments by Alvarez *et al.* [3]. In particular, their sources were stationary and symmetric, while their sinks separated regions of different k and exhibited phase matching motion. It was noted by them [3] that phase matching was inconsistent with a Ginzburg-Landau description of the dynamics, since phase matching involved both fast and slow length scales, while the Ginzburg-Landau equations are derived by explicitly removing fast scales.

Goldstein *et al.* [14] have studied defects in traveling waves near a parity-breaking transition. They also used a Ginzburg-Landau-type model which one would not expect to be valid far above the transition. They predicted that above the transition, both sources and sinks would be stable, but that close to the transition one or the other type of defect would be unstable. We observe sources to be stable, while sinks are transients. In most cases, sinks were so unstable that they survived only a few seconds after being produced by perturbing the system, and could not be studied in any detail. The defects studied in the simulations of Ref. [14] were symmetric and stationary, while our sinks are not symmetric and move.

We observed that instabilities at the boundaries and in the bulk of the pattern affected the defect dynamics. Sinks exhibited sudden phase jumps, whereby they moved by one wavelength and absorbed an extra finger. Perturbations at the boundaries propagated in towards the sinks, sometimes resulting in a wavelength-changing Eckhaus instability in the bulk of the pattern and causing changes in the motion of the sinks. Sources, on the other hand, while stationary on average, exhibited irregular motion which appeared to be driven by experimental noise.

V. CONCLUSIONS

We have experimentally studied the behavior of sink and source defects in traveling finger patterns in the printer's instability. Both could be created by perturbing the finger pattern. We found sinks to be transients. In most of the parameter range studied, sinks survived only briefly. Relatively long lived, though still transient, sinks could be created only in a range of v_o^* far above the parity breaking transition at which the traveling fingers form. Sinks separated regions of different wave number and moved smoothly to keep the phase relationship between the pattern on the two sides of the defect constant. Sources, in contrast, were stationary and symmetric on average. Individual sources moved erratically, apparently driven by experimental noise.

Theoretical treatments of source and sink behavior in traveling wave systems have been based on Ginzburg-Landau models. These models are expected to be valid only rather close to onset. While some of the behavior we observe is consistent with the theoretical models, much of it is not. In particular, the phase-matching dynamics exhibited by our sinks, and observed previously in other experiments [3], cannot be explained within the Ginzburg-Landau approach.

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